# Anoka-Hennepin Secondary Curriculum Unit Plan

| Department: | Mathematics | Course: | Intermediate Algebra | Unit 4 Title: | Situations That Can Be Modeled with Quadratic Functions | Grade Level(s): | 9 |
|---|---|---|---|---|---|---|---|
| Assessed Trimester: | Trimester A | Pacing: | 18-20 Days | Date Created: | 6/13/2013 | Last Revision Date: | 7/26/2014 |

**Course Understandings**: *Students will understand that:*

A. Relationships exist between real-world situations, mathematical equations, inequalities and graphs for linear, exponential, absolute value, radical, and polynomial functions.
B. Equations and inequalities can be categorized by form and that each form has specific processes to consider when solving and graphing.
C. There are a variety of strategies of varying efficiency for simplifying linear, absolute value, exponential, radical, complex and polynomial expressions.
E. The context of a problem is important in recognizing the reasonableness of a solution.
F. There are benefits and limitations in the use of calculators and other technology to solve mathematical situations.

## DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

### Established Goals

**Minnesota State/Local/Technology Standard(s) addressed (2007):**

- **Standard (9.2.1.# - Modified):** Identify important features of Quadratic functions and other relations using symbolic, tabular and graphical methods where appropriate.
  **Benchmark:**
  **9.2.1.3** Find the domain of a function defined symbolically, graphically or in a real-world context.
  **9.2.1.4** Obtain information and draw conclusions from graphs of functions and other relations.
  **9.2.1.5** Identify the vertex, line of symmetry and intercepts of the parabola corresponding to a quadratic function, using symbolic and graphical methods, when the function is expressed in the form $f(x) = ax^2 + bx + c$, in the form $f(x) = a(x - h)^2 + k$, or in factored form.

- **Standard (9.2.2.# - Modified):** Recognize quadratic functions in real-world and mathematical situation. Represent quadratic functions with tables, verbal descriptions, symbols and graphs. Solve problems involving quadratic function and explain results in the original context.
  **Benchmark:**
  **9.2.2.3** Sketch graphs of linear, quadratic and exponential functions, and translate between graphs, tables and symbolic representations. Know how to use graphing technology to graph these functions.

- **Standard (9.2.3.# - Modified):** Generate equivalent algebraic expressions involving polynomials; use algebraic properties to evaluate expressions.
  **Benchmark:**
  **9.2.3.2** Add, subtract and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.
  **9.2.3.3** Factor common monomial factors from polynomials, factor quadratic polynomials, and factor the difference of two squares.

- **Standard (9.2.4.# - Modified):** Represent real-world and mathematical situations using equations and inequalities involving quadratic functions. Solve equations and inequalities symbolically and graphically. Interpret solutions in the original context.
  **Benchmark:**
  **9.2.4.1** Represent relationships in various contexts using quadratic equations and inequalities. Solve quadratic equations and inequalities by appropriate methods including factoring, completing the square, graphing and the quadratic formula. Find non-real complex roots when they exist. Recognize that a particular solution may not be applicable in the original context. Know how to use calculators, graphing utilities or other technology to solve quadratic equations and inequalities.

### Transfer

**Students will be able to independently use their learning to: (product, high order reasoning)**

- Analyze a real world situations using quadratic functions and make predictions.

## Meaning

| Unit Understanding(s): | Essential Question(s): |
|---|---|
| **Students will understand that:**<br>● Rate of change can be used to discuss the difference between linear, exponential and quadratic functions.<br>● Significant features of quadratic graphs can be used to analyze real world situations and solve real world problems. | **Students will keep considering:**<br>● How do I know what quadratic form (factored, vertex, standard) of the function is best for a given situation/question?<br>● When looking at quadratic functions, how do the significant features of the graph, its algebraic equation and real world representation relate to each other?<br>● Where can I find situations of quadratic functions in the real world?<br>● How do the skills and knowledge that we are learning influence the task of making decisions and planning... a rocket launch, the height of roof for a new football stadium, etc? |

## Acquisition

**Knowledge - Students will:**
- Know how many solutions a quadratic equation has.
- Demonstrate understanding of the relationship between different forms of quadratic equations and their graphs.
- Understand the definition of a quadratic function and the vocabulary of the significant features of a graph.
- Understand the definition of a domain and range

**Reasoning - Students will:**
- Demonstrate understanding of the significant features of a graph of a quadratic function and their relationship to real-world situations (vertex, line of symmetry, intercepts, domain and range)
- Understand certain functions have a restricted domain
- Draw qualitative conclusions based on the graphs
- Interpret quadratic inequalities to solve real world situations

**Skills - Students will:**
- Identify the significant features of a graph of a quadratic function (vertex, line of symmetry, intercepts, domain and range, rate of change)
- Graph quadratic functions
  - Standard form, vertex form, and factored/intercept forms
  - With and without a graphing calculator
- Translate quadratic equations from factored and vertex forms INTO standard form.
  - Multiply binomials
- I can translate quadratic equations from standard form INTO factored or vertex form.
  - Factor a quadratic expression
  - Complete the square
- I can use tables and graphs to solve quadratic equations including real world situations and translate between representations.
- I can graph quadratic inequalities
- Interpret a solution in the original context
- Compare solutions to appropriate graphical or numerical estimates
- Use a graphing calculator to justify the equivalency of 2 expressions

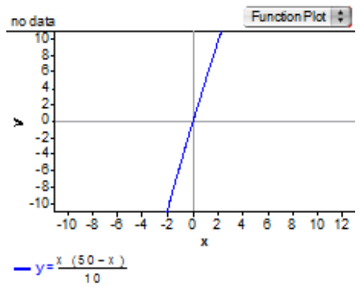
**Common Misunderstandings**

- 'Graph as a picture' confusion occurs when students believe that the graph of a relationship between two variables is the actual picture of the situation. For example, suppose a ball is thrown straight up in the air and allowed to drop to the ground. The relationship between the time in the air and height of the ball is a parabola but the actual path of the ball follows a vertical line. Many students think that the path of the ball follows a parabola when actually only the relationship between time and height is parabolic. This problem also shows up when students are asked to sketch the graph of height of a person riding on a Ferris wheel as a function of time. Many students will draw a circle to represent the relationship rather than the periodic function that it is.
- Students struggle with function notation, specifically the *f(x)*. Some students think that *f(5)* means a variable *f* multiplied by 5 rather than knowing that *f(5)* is the value of the dependent variable *y* when the value of the independent variable *x* is 5.
- Students interchange the x-intercept and the y-intercept
- Students misunderstand the zero of a function to mean when x = 0, rather than when y = 0.

**Essential new vocabulary**

- Axis of symmetry
- Coefficient
- Completing the square
- Degree
- Difference of two squares
- Factor completely
- Factoring
- Like terms
- Maximum
- Minimum
- Polynomial expression
- Perfect square trinomial
- Quadratic equation
- Roots of a function
- Term
- Vertex
- Zeroes

- The domain of most quadratic functions is the set of all real numbers but many students lack the arithmetic sense to compute with numbers that are not integers. Students may be able to graph the function $f(x) = 3x^2$ using integral values for x but would not be able to calculate the exact value of $f(21)$.
- Students state that x and y=-intercepts are values rather than the coordinates of points on a graph. The y-intercept of the line y = 2x + 7 is the coordinate (0,7) not the value 7. The x-intercepts of the function y = 3(x + 5)(2 - x) are (-5,0) and (2,0) not -5 and 2. This is an important distinction since intercepts highlight important features of the relationship between two variables and not simply the value of one variable.
- Students incorrectly identify the features of a function based on limited information. For example, a student might incorrectly explain that the graph of the function $y = \frac{x(50-x)}{10}$ is a linear function because when the function is entered into a graphing calculator using the default settings the graph looks like a line.




- Students will incorrectly state that the maximum value of the function $y = -3(x - 11)^2 + 7$ is 11. Students confuse which variable, the independent or dependent, is being maximized or minimized and which variable determines where this occurs.
  Students use end behaviors of a function within a specific graphing window to represent the maximum or minimum values of the function.
- Students incorrectly subtract polynomial expressions as shown below:

$$(3x^2 - 5x + 7) - (x^2 - 3x - 2)$$
$$3x^2 - 5x + 7 - x^2 - 3x - 2 \quad \text{(not an equivalent expression)}$$
$$2x^2 - 8x + 5$$

- Students often struggle to create a mathematical model for a real-world situation.
- Students confuse the meaning of exponents and incorrectly calculate the value of exponential expressions $(3a)^2 = 3a^2$
- Students incorrectly apply the distributive property to multiply polynomials. (e.g., $(3a + b)^2 = (3a)^2 + (b)^2$).
- Students will neglect partial products when multiplying polynomials. For example, some students incorrectly simplify the expression $(x - 3)^2$ by writing $(x)^2 + (-3)^2$ and ending up with $x^2 + 9$ for an answer. Some students incorrectly apply a memorized "FOIL" procedure when simplifying expressions like $(x^2 - 3x + 9)(x + 2)$
- Students incorrectly state that graphs of quadratic functions have vertical asymptotes.